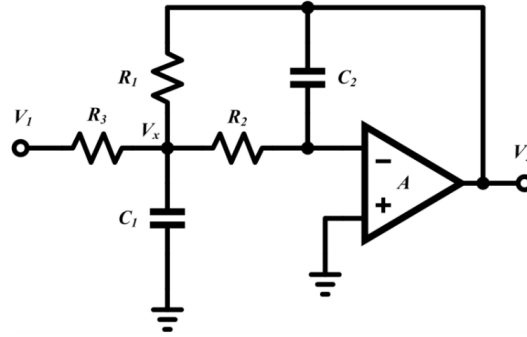


# ECE580

## HOMEWORK 5



$$H(s) = \frac{G_2 G_3 / (C_1 C_2)}{s^2 + s(G_1 + G_2 + G_3)/C_1 + G_1 G_2 / (C_1 C_2) + \varepsilon}$$

1. Characteristic equation for a Second order filter is  $s^2 + s \frac{\omega_0}{Q} + \omega_0^2 = 0$ .

$$\omega_0^2 = \frac{G_1 G_2}{C_1 C_2}, \quad \omega_0 = \sqrt{\frac{G_1 G_2}{C_1 C_2}}$$

$$H(0) = \frac{G_3}{G_1}$$

$$\frac{\omega_0}{Q} = \frac{G_1 + G_2 + G_3}{C_1}, \quad Q = \frac{\omega_0}{(G_1 + G_2 + G_3)/C_1} = \frac{\sqrt{G_1 G_2 / (C_1 C_2)}}{(G_1 + G_2 + G_3)/C_1} = \frac{\sqrt{G_1 G_2}}{(G_1 + G_2 + G_3) \sqrt{C_2}}$$

2.  $Q^2 = \frac{G_1 G_2}{(G_1 + G_2 + G_3)^2} \frac{C_1}{C_2}$

$$\frac{C_1}{C_2} = Q^2 \frac{(G_1 + G_2 + G_3)^2}{G_1 G_2} = Q^2 \left( \sqrt{\frac{G_1}{G_2}} \left(1 + \frac{G_3}{G_1}\right) + \sqrt{\frac{G_2}{G_1}} \right)^2$$

$$\text{Let } x = \sqrt{\frac{G_1}{G_2}}, \text{ then } \frac{C_1}{C_2} = Q^2 \left\{ x \left[1 + H(0)\right] + \frac{1}{x} \right\}^2$$

$$\frac{d(\sqrt{C_1/C_2})}{dx} = Q \left\{ [1 + H(0)] - \frac{1}{x^2} \right\} = 0, \quad x = \frac{1}{\sqrt{1 + H(0)}}$$

$$\min \left( \frac{C_1}{C_2} \right) = Q^2 * \left( 2\sqrt{1 + H(0)} \right)^2 = 4Q^2(H(0) + 1)$$

3.  $\frac{C_1}{C_2} = Q^2 \frac{(G_1 + G_2 + G_3)^2}{G_1 G_2}$ , if all resistors have the same value, i.e.,  $G_1 = G_2 = G_3 = G_0$

$$\frac{C_1}{C_2} = Q^2 * 3^2 = \left( 2^{-\frac{1}{2}} \right)^2 * 3^2 = \frac{9}{2}$$

$$\text{According to 2, } \min \left( \frac{C_1}{C_2} \right) = 4Q^2(H(0) + 1) = 4 * \left( 2^{-\frac{1}{2}} \right)^2 * (1 + 1) = 4.$$

**So  $C_1/C_2$  changes from 4 to 9/2, by 12.5%.**